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Constructivism and Constitutionalism: Some Implications for the First Mathematics Education (*)

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We take our point of departure in Hermain Sinclair's (1988) and Ernst von Glaserfeld's (1988) recent discussion of constructivism in relation to early mathematics education. As both authors deal with some central themes within constructivism, we will make a few general comments first and thereafter briefly develop some of the implications for mathematics education in the early school years.

To begin with we would like once again to reflect briefly on the core meaning of constructivism, especially in relation to the qualified form «radical constructivism». Sinclair declares that she uses the term «constructivism» in the sense of «Piagetian constructivism», and she explains that constructivism sees our mental or material actions as the main source of knowledge. This is in contrast with the empiricistic notion that knowledge originates from «the outer reality», through our senses impinging on our minds.

von Glasersfeld locates constructivism in a wider intellectual context (of rationalism) and refers to the early 18th century philosopher Giambattista Vico as the chief inspirer. «Radical constructivism» stands for — in von Glaserfeld's use of the term — the view that our knowledge is always confined to the world of human experience and that it cannot be judged against the criterion of how well it matches the world as it is; at best our knowledge will fit our pursuit or goals.

According to von Glasersfeld not even the most radical form of constructivism would, however, deny the existence of a reality independent of the experiencing human beings. It is just that this «real» reality is out of our reach, it is not available for rational knowledge.

It is exactly this demand that our search for objective truth should be abandoned that triggered some rather hostile comments during the 1987 Psychology of Mathematics Education meetings (see Kilpatrick, 1987; Wheeler, 1987).

Our own position coincides with neither that of constructivism, nor that of realism (according

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to which true knowledge mirrors what the world is really like). We agree very much with the constructivist stance, and especially with von Glasersfeld's emphasis on the world-as-experienced as the realm of human thought. But, for the economy of the argument, coupled with the need to give some background to what will follow, we will restrict ourselves to mentioning just one point of fundamental disagreement.

1. THE NATURE OF HUMAN-WORLD RELATIONS

The main problem with constructivism (including radical constructivism) is, in our view, that the individual and the world are seen as separated from each other. This leads to paradoxes. For example, according to the constructivist way of thinking, the individual can never get in touch with the reality that he is divorced from. As all knowledge is assumed to be derived from the individual's constructing activity it is very difficult to see how he can find out about the constraints imposed by the surrounding world that would lead to accomodation. After all, the constraints experienced must be — according to the constructivist assumption — *constructed* constraints and these must be formed in accordance with the properties of already given scheme activity. As Feffer (1988) points out:

> ... the subject can only adjust his receptors to constructed «shapes», constructed «demands» and, indeed, constructed «feedback». In terms of the constructionist proposition, then, the reality that serves to correct an assimilatory construction can only be rendered as another construction by an already given scheme, and we are no closer to the «real» object than when we started. In sum, there can be no last court of appeal to an independently constituted reality in the course of holding to the constructionist proposition, for the judgment handed down will iltself turn out to be the result of a distorting construction. (3: pp. 18-19)

An alternative to the epistemological (and ontological) position of constructivism is based on the principle of *intentionality*, one of the cornerstones of the phenomenological tradition. According to this principle all mental acts are directed towards something, something beyond themselves. To think, for instance, is always to think something and to perceive is always to perceive something. The individual's experience of the world is a relation between the individual and the world, both are presupposed. Thus there are not two separate entities (individual and world) plus a relation between them; the world-as-experienced is all there is. The world we know, experience, think about must necessarily be a world known, experienced, thought about. Quite obviously, we cannot think of what a world in itself, a world not thought about, would be like. And a known, experienced, thought about world presupposes a knowing, experiencing, thinking subject. Likewise, the acts of knowing, experiencing, thinking presuppose a world to know, experience and think about. It is, however, not a world only inside our heads, it is a real world «out there». It can be experienced in different ways, of course, but in relation to certain criteria we can surely argue that one way of experiencing it (or understanding it) is better (in the sense of being deeper, or more efficient, or the like) than another. The claim that there is no independently constituted reality, or that there is no absolute knowledge of transcendental nature, means simply that we can in no way come up with a finite description of anything. Phenomena are fundamentally inexhaustible as far as possible descriptions of them are concerned. Some modes of experience are shared by a culturally ou professionally defined group of people, others of all human beings, yet others of all mammals and so on. But we can never claim that there are no other ways of seeing a phenomenon than the ones that we know of. In this sense the world of human beings includes the human beings themselves. An experience always takes someone to do the experiencing and something to be experienced; the experience comprises a relation between them. This is why this school of thought can be called constitutionalism.

The next thing we would like to do is to link this general line of reasoning with some implications for mathematics education. In order to do so let us briefly consider the Piagetian distinction that appears in Hermaine Sinclair's paper, between empirical and reflective abstraction. The former refers to the abstraction of «the properties of objects» and the latter to the abstraction of «one's own action and reasoning coordination». Logic and mathematics is said to be based on the second kind of abstraction, and we can also think of it as being a higher, second order abstraction in comparison with the first kind of abstraction. Sinclair gives two examples: She argues that «weight» as an object of thought needs both kind of abstractions, «number» as an object of thought, on the other hand, only needs the second kind.

Now, one might object that thinking about numbers also reflects some aspect of the experienced world and not only of the experiencing individual. Numbers are also human-world relations. This is a highly relevant point for exemplifying how the difference between a constructivist and a constitutional view may show itself in differing interpretations of what it takes to develop elementary arithmetic skills.

2. DOUBLE COUNTING

Steffe, von Glasersfeld, Richards and Cobb (1983) have described, within a constructivist frame-work, five increasingly sophisticated types of units that children create when they count. The fifth and most advanced of these counting types is supposed to illustrate the second kind of abstraction named by Sinclair. The concrete criterion set up by Steffe et al. for inferring that a child has reached this level, is the ability to count counting words:

> ... we infer this final accomplishment when the child spontaneously uses «double counting» ... for example, if the problem is «to count on seven from nine» and the child says: «9 ... 10 is 1, 11 is 2, 12 is 3 ... 16 is-sixteen» (p. 43).

Our own understanding of the significance of double counting is distinctly different. Prior to the main investigation, which we are going to describe later, a pilot study was conducted in which it was observed that the most salient difference between pupils with specific mathematics difficulties and their class mates was specifically that the former group of children could use no problem solving strategies other than «double counting» (Eriksson & Neuman, 1981). Their difficulty seemed to be that they had found no other methods for finding out the size of the added or subtracted term if it was greater than the subitizing range (i.e. the range for number units that we can perceive immediately, usually up to 3). They tried to «hear» its size through enumerating the number words corresponding to the units in it. If the number of words was still not perceptible, they had to count them by keeping track in some way.

Their «double counting» was rarely done in the way described by Steffe et al., who describe children saying, for example, «Three is one, four is two ...» etc., but they counted the words in the added or subtracted term on their fingers. Thus one finger was raised for each word enumerated, e.g. in the task 9-6, the pupil would raise one finger for each of 8, 7, 6, 5, 4, 3, and recognize that raising the sixth finger meant that counting should stop.

The problem with the «double counting» method is that it does not lead to functional arithmetic skills. As soon as there are more than three counting words they have to be counted in some way. This is an effective barrier to developing the skill of mental calculation, and thus also of estimation. Further, it prevents the child from learning the multiplication tables and thus also carrying out divison.

When the pupils with mathematics difficulties got multiplication tasks to solve, they calculated, for example, 7x8 by saying ... 16 ... 17, 18, 19, and so on, raising a finger for any word enumerated. However, they fairly soon lost track of the number of *times* they had enumerated the eight words they counted each time on their fingers. In order to solve that task they would need to be able to «triple count» in some way.

Even in the highest grades in the compulsory school such pupils found it necessary to resort to «double counting» even to solve some simple subtraction and missing addend tasks within the number range 1-10, as the difficulty had less to do with the size of the whole number than with the size of the part; if the first word is not «one», the last word does not give the answer. It was this part that always had to be «double counted» if it was larger than «3».

That they had not acquired what are usually called the «basic number facts» — not even within the number range 1-10 — was another of the salient traits in the arithmetic skills — or rather lack of skills — of the children with mathematical difficulties.

However, we do not use wish to the term «number facts» when referring to, knowledge of the part-whole combinations within the number range 1-10, since we do not think of this knowledge as «remembered facts» but as a strongly conceptual kind of knowledge. We have proposed that the knowledge of the part-whole combinations of the first ten numbers should be referred to as the knowledge of the «ten basic concepts».

When the child can divide up the ten first positive numbers into two parts, each of these building stones in the decimal system has become a concept, possible to think of in many different ways. For example, 8 can be thought of as 2/6, 4/4, 3/5, etc. and also as a part of all greater numbers. As the numbers 2 and 6 are also concepts, 8 thought of as 2/6 could further be thought of as 1/1/3/3/ or 2/2/3/1, etc. (In the jardon of phenomenology this is tantamount to grasping the essence of 8)[1]. All the numbers within the first decade have become a tight «network» of relations within and between numbers when the «ten basic concepts» are formed.

As the pupils with mathematics difficulties had not acquired the «ten basic concepts», they could not solve all different kinds of problems — within and outside the number range 1-10 — using economic analyzing strategies, as the other pupils could. For example, these other pupils could analyze the quantitative relations inherent in the problem in order to see if it were possible to reduce the problem to one involving a multiple of 10 (for example «82—7=75 since if you first take 2 away from 7 and 82 simultaneously you get 80—5=75»). They could also analyse the quantitative relations in order to observe if it were possible to group into doubles plus-or-minus one (for example 13-6=7 since 6+6=12), or as doubles with units moved from one oart to the other (for example 6+8=14 because 7+7=14), a strategy which, however, can only occasionally be used.

In other words, while Steffe et al. (1983) refer to double counting as a concrete manifestation of the most advanced counting type, Eriksson & Neuman (1981) observed double counting as the dominant strategy used by children with mathematics difficulties. This led them to inquire into how the majority of children manage to solve simple arithmetic problems *without* double counting. Because they had conclued that mastery of the ten basic concepts is necessary, an investigation was carried out with the aim of revealing how children develop this competency, fundamental for arithmetic skills.

3. A PHENOMENOGRAPHIC APPROACH TO RESEARCH

It was mentioned earlier that we prefer to talk about «basic concepts» rather than «number facts». This is because we put the emphasis on how children see (or apprehend, or conceptualize) numbers and number relations instead of asking what facts they may be able to retrieve from their long term memories. Such a perspective is very much in accordance with the basic tenets of the research approach called phenomenography (Marton, 1981) that has been developed in our research group in Gothenburg.

Phenomenography originates from the observation that whatever phenomenon people encounter, there seems to be a limited number of qualitatively different ways in which that phenomenon is seen, experienced or conceptualized. The aim is to reveal and describe the differing conceptions, differing understandings of the various phenomenon in the world around us. Furthermore it is assumed that our way of conceptualizing a certain phenomenon is the most fundamental aspect of our knowledge about our skills related to that phenomenon (Marton, 1988).

[1] We would like to thank professor Amedeo Giorgi of the Saybrook Institute, San Francisco, CA, for this and other comments on an earlier version of this paper.

While Piagetian constructivism has a clear psychological orientation, the constitutional frame-work — to which phenomenography clearly belongs — is more easily reconcilable with didactic considerations. While the emphasis in constructivism is on acts — material or mental — constitutuionalism has the unity of the act and that which is acted upon as its point of departure. This latter stance makes it fairly natural to describe different ways of thinking about a certain phenomenon, or different ways of dealing with that phenomenon, in relation to the particular competency we aim to develop in an educational setting. The differences can then be seen as increasingly functional human-world relations.

In the study we will draw on here (Neuman, 1987) one hundred and three 7-year-old school starters were interviewed about problems set in words, involving the number range 1-10, such as, for instance:

> If your teacher has only two pencils in her box and there are nine children wanting to make drawings, how many pencils does she heve to get? («2+—=9»);
> If you have only 3 crowns and you want to buy a comic for 7 crowns, how many more crowns do you need? («3+—=7»);
> If your teacher has only four pencils in her box and there are ten children wanting to make drawings, how many more pencils does she have to get? («4+—=10»).

The children were asked about how they had «thought out» their answers, if it had not been possible to observe overt counting. The idea was that it might be possible to find strategies that these, as yet untaught, school starters had begun to use which were teachable and would result in the acquisition of the «ten basic concepts».

Of the more than one thousand answers given by these 103 school starters, only a total of 12 children altogether gave 17 answers where some kind of «double counting» was used in «counting on» strategies.

4. ALTERNATIVES TO DOUBLE COUNTING

How, then, did the children in Neuman's study solve the problems?

Those of them who used strategies which yielded correct answers used some of three types of strategies, all of them analyzing (or structuring) — not counting — strategies. Thus they did not «count out» - or unitize one unit at a time — but they analyzed the quantitative relations in the problems — exactly as the older pupils who had no mathematics difficulties had done in the earlier study. They used three such analyzing strategies which will now be described.

4.1. *The first strategy*

Was very similar to those which Resnick (1983) has called «choice» and «min». Neuman has called it «Biggest first» (1983). The main idea in these strategies was that the name of the first unit in the largest part should be «one». In contrast with Resnick's account, Neuman found these strategies in «counting all» as well as in «counting on» strategies, in open sentences and subtraction as well as in addition.

Jonas, a boy who could give practically no correct answers in the interviews when he started school, had within six weeks — in a follow-up interview — found this principle, and could also describe how it had been created. He was asked how many more pencils a teacher would need if she had two in her box but nine children came to the class. He solved the question «2+—=9» in the following way:

> J: ... seven.
> I: How did you know that?
> J: Well, because I've done it on a piece of paper.
> I: You didn't... you just sat and looked at the ceiling!
> J: No, but I've worked it out at home!
> I: And you remember that? How did you work it out then?
> J: Like this... first I took two (drawing in the air)...
> I: You can have this piece of paper if you like!
> J: I took two (draws two lines to the left of the paper) like this.
> I: Mmm...
> J: And that made... (moves the pencil a bit to the right and starts drawing one line at a time from to left)... one, two, three,

four, five, six, seven, eight, nine... oops... I did it wrong... (rubs out the last two lines he had drawn).

I: *So you counted forwards to one, two,* three, four, five, six, seven now... and you knew that they (point at the first two lines) still were there?

J: Mmm...

I: That's neat!

(Neuman, 1987, p. 163)

4.2 *The second strategy*

used was to analyze the whole number in order to see if it could be divided up into two similar parts, i.e., in order to see if the «doubles» could be of some use.

4.3. *The third strategy*

was to analyze a «finger number» in order to see if it was possible directly to subitize the two parts within the whole. The ways in which the children used this strategy depended on whether they knew these «finger numbers» or not. If they did not know them, they first had to create them by giving names to each of their fingers. One of the children — Susie — solved the question «3+—=7» in this way. After giving names to her seven fingers — «1, 2, 3, 4, 5, 6, 7» — she said:

— There... (watching her «finger number»)... now we can start!

There... was now the «finger number» created, and she could begin the analyzing process. Susie, who neither knew the «finger number» for nine (nor that for seven in the question «2+—=9»), started by creating the «finger number» for nine. She then solved the problem in the following way:

S: She had to go and get seven pencils.

I: So, first you counted nine fingers... what did you do after that?

S: First I sort of counted five... Then I put up two... Then I put put up two more... Then I put them down (the two last of the nine fingers) and counted them again like this... (counted the seven fingers on her lips) (op cit, p. 188).

Just like Jonas, Susie seemed to understand in some way, that if the known part is as small as two, the other part cannot be subitized (i.e. its numerosity cannot be perceived immediately). However, as she watches her «finger number» she discovers the idea of how to solve this problem in a different way from Jonas: she discovers that it is possible to group this part into a subitizale «5 + something» pattern. Therefore, without consciously knowing why, she «sort of counted five», i.e. the whole hand, and so put up two fingers more, up to the two fingers in the known part, which thus became the last fingers in the «finger number», exactly as the pencils became the last part of the pencils Jonas drew.

Susie's «finger number» is similar to the ancient Roman number for nine, which was a picture of the hand and four fingers: VIIII. Exactly as this number has to be divided up in the parts VII/II if the V-symbol is not to be split up, Susie's fingers are divided up in this way in order not to split up the hand.

Her idea of grouping the large unknown part in a way that makes it possible to subitize involves the same important idea as the one created by Jonas, that the name of the first unit in the large part must always be «one». Then both parts can immediately be «subitized» within the whole. In Susie's case this depends on three things, on the large part of the «finger number» being grouped, on the name of its last finger denoting the whole number, and on the small part then becoming small enough to subitize.

In order to solve the problem in a more economic way, Jonas would have had to be able to count backwards: «9, 8... 7». It was possible to observe how difficult this was for children who had not created «finger numbers». They did not seem to trust the first word they arrived at in the unknown part as a word denoting this part; it was usually the last word that had that function. Thus, if any of the above tried counting backwards, they counted to «1» and then back again to the highest ordinal number of that part. This kind of counting backwards, however, was not carried out in the word problems. It was only observed in some unanalyzed tasks where some of the objects were visible.

Susie had to give names to her fingers in order to know where the «finger numbers» ended. However, when the «finger numbers» are known, children can solve the problem in the way Gail does. She just throws a glance at her fingers, and answers «seven» to the question «2+—=9». She has given names to all her fingers and illustrates for the interviewer that the ring finger of her right hand is «nine» and that the second forefinger is «seven». It is enough for her just to look at her concrete «row of finger numbers».

However, this concrete «row of finger numbers» rather soon seems to be imagined again. Niclas has illustrated how the problem is then solved. Niclas has got the question «4+—=10» and answers in this way:

N: Six.
I: She has to fetch six... how did you know that?
N: How many children did you say...?
I: I said that she had ten children and four pencils.
N: Yes, because you should put one to four (illustrates with his fingers how the thumb of the second hand is unitized with the four fingers on that hand) and a five here (the thumb of the first hand is added to the second hand; thus the «finger number» 4/6/10 is formed).
I: ??? (The interviewer neither understands what Niclas says, nor what he does. So she asks again:) Show me... you put your fingers...
N: There are four like this... (the four fingers on his second hand).
I: Yes... and then...?
N: Then I take six (the thumb on his second hand)... I mean then I take one there... (op cit, p. 195).

Niclas is illustrating how he has changed his known «finger number» 5/5=10 into 6/4=10 in his thoughts, by moving one thumb over from one hand to the other. He illustrates how his separate fingers have become names (the thumbs, for example, being called «5» and «6»), but also that the finger group 5+1 can be called «6» and that the thumb called «6» also could be denoted «one».

5. HOW QUANTITATIVE ASPECTS INHERENT IN THE PROBLEMS APPEAR

While the ways in which the children dealt with the problems in the investigation were being observed, it could also be observed how certain meanings were constituted, how certain quantitative aspects inherent in the problems became visible to the children.

When Susie tries to get hold of how many units there are in the unknown part in question «2+—=9» the fingers suddenly reveal their «fiveness», making it possible for Susie to make the large part perceptible by subitizing. For Jonas the row of drawn pencils shows that either of the pencils at the endpoints could be given the name «1».

We can also imagine how the pairs and doubles reveal themselves in dicepatterns, in the «two+two» legs of animals etc.

One of the children, Amanda, actually in the middle of a problem situation also expresses how the world, even if this worls is a part of her own body, insists on taking part in solving the problem. Having failed to answer some of the questions she suddenly says:

— Let's see... I'll count on my fingers... they want me to... (op cit, p. 147)

This girl did not even know that she had ten fingers when the fingers «wanted her to use them»; she had to count them first. Still — in the last question she solved, she had already begun to understand how to use them.

From time to time it is thus illustrated that the child does not consciously reflect on how to solve the problem. He or she simply explores the quantitative relations inherent in it, and suddenly something stands out as a figure against a background, giving the child an intuition of how it might be solved.

Thus, we claim that children's problem solving strategies are actually formed when the quantitative relations in the problem become visible to them. Further, we claim that the skills for solving quantitative problems are not counting skills, but rather «seeing skills», analytic skills. Children begin to analyze and reflect on known numbers, e.g. «doubles» and «finger numbers» to which they have given their «numerical names» by counting — or rather by enumeration — and they later analyze «imagined numbers» of this kind. They also contemplate how they can transform them in *different* ways *into* new part-whole relations. Niclas, for instance, illustrates how 5/5=10 can be transformed into 4/6=10.

Thus we argue that it is the children's seeing of certain quantitative relations that enables them to solve simple arithmetic problems. Revealing what quantitative relations they ought to see or discover is obviously of didactic relevance. It is reasonable to believe that such insights would increase our possibilities for making these relations visible to them. However, one more step is needed: the strategies used by the children have to be compared with regard to how functional they are in relation to the goal of mastering the «ten basic concepts».

Of course, «fiveness» also reveals its force when the fingers are used to count the words in «double counting». However, this «double counting» strategy demands that counting is carried out twise, simultaneously, — with the words and with the fingers — so that its use leaves insufficient cognitive resources for more complex problem solving. This is why pupils who have not found any other strategies end up with difficulties in mathematics. The use of «finger-numbers», on the other hand, leads towards mathematical thinking, since it is the fingers themselves that both count and group the numbers; and all that is done without any counting — just by analyzing.

The reason why the strategy «double counting» as described by Steffe et al. (1983) is seen by them to be the most advanced counting type, and by us to be a strategy leading to mathematics difficulties, thus depends, at least in part, on the fact that we are more interested in how functional the strategy is and less in its degree of abstraction.

6. CONCLUSIONS

Our aim in this paper has been to compare the constructivist paradigm with an alternative frame-work, here called constitutionalism. Above all we wanted to show how certain differences appear in views on the development of arithmetic skills.

The constructivist paradigm puts the emphasis on the individual's acts, while within a constitutional frame-work we are primarily interested in how various aspects of the world are seen by different individuals. It is only against such a background-it is argued-that we can understand the individual acts.

We have claimed that the constitutional alternative lends itself more easily to a didactic «knowledge interest»: finding out about the implications of different ways of seeing a phenomenon should reasonably give us clues to what aspects of that phenomenon we should try to make visible in teaching.

It has been suggest, however, that there is a more fundamental difference between the two alternatives. According to the idea of constructivism the individual cretaes his own world. It is a subjective world which is different and divorced from the real world which is simply available to us. According to the idea of constitution, a cornerstone of «the phenomenological movement», the individual and the world form a unity, we live *in* the world, a world which is an experienced, a lived in, a thought about world. It is both objective and subjective, a real world, the only world we have.

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ABSTRACT

There is a dualistic assumption underlying constructivism: thinking takes place in an inner subjective world, divorced from the outer objective reality and knowledge is constructed there by the individual through material and mental acts. In a phenomenological framework the fundamental unity between human beings and the world in which they live is assumed. Knowledge represents ways of seeing, experiencing, thinking about the world and it is constituted through the internal relation between the knower (subject) and the know (object). It is shown that these two kinds of ontological and epistemological assumption have radically different implications for how the development of arithmetic skills is seen and conceptualized.